

BRIEF COMMUNICATION

RISE VELOCITY OF LARGE BUBBLES IN STAGNANT LIQUID IN NON-CIRCULAR DUCTS

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Abstract—A method for estimating the rise velocity of large bubbles in non-circular vertical ducts is proposed. It is suggested here that the bubble rise velocity is controlled by the effective average radius of curvature at the bubble cap. This curvature is estimated on the basis of the suggested bubble shape, which is related to the duct geometry.

The rise velocity of large bubbles in unbounded media and in channels has received considerable attention both experimentally and theoretically. In the analysis of Davis & Taylor (1950) the rise velocity of free spherical capped bubbles in stagnant liquid was considered. The surface tension and viscous forces were neglected and the bubble motion was assumed to be determined by the free fall of the liquid along the upper surface of the bubble. They found that the rise velocity, V_b , of an unbounded bubble was directly proportional to the square root of the local radius of curvature at the top of the bubble:

$$V_b = C\sqrt{gD}, \quad [1]$$

where D is the curvature diameter (twice the radius of curvature), g is the acceleration due to gravity and $C = 0.47$.

Davis & Taylor (1950) showed that [1] is applicable also for restricted bubble flow in round pipes, in which case D in [1] is the pipe diameter, and $C = 0.35$.

Only a few studies dealing with the bubble rise velocity in non-circular channels have appeared. Geometries, such as rectangular ducts, annuli as well as rod bundles are of interest in industries concerning with nuclear plants, heat exchangers and gas-oil risers.

An intuitive approach for calculating the bubble rise velocity in geometries other than a round pipe would be to use [1] with the same constant $C = 0.35$, but with a proper equivalent diameter D_e . The choice of a hydraulic diameter as the equivalent diameter has been proven to be totally wrong, giving in many cases substantially lower values than those observed experimentally. Griffith (1964) suggested using the large dimension of the channel as the characteristic diameter in [1]. His suggestion was partially successful, but still the experimental value of C appeared to depend considerably on the channel geometry.

Recently, Sadatomi *et al.* (1982) injected solitary air bubbles into stagnant water columns of different geometries and measured the bubble rise velocity. They suggested using an equi-periphery diameter, which is equivalent to the wetted periphery divided by π . Using this characteristic dimension, they correlated their data as well as that of Griffith (1964), applying [1] as suggested by Davis & Taylor (1950). The constant C was found to be in the range of 0.31–0.39. Although this selection of D_e seems to be successful, no physical explanation was given for the choice of the equi-periphery diameter as the characteristic dimension.

In order to base any possible explanation on physical grounds we recall that in the original derivation of Davis & Taylor (1950) the bubble rise velocity depends on the radius of curvature of the bubble's cap at the top. Thus, it is assumed here that the rise velocity of bubbles in non-circular ducts is also determined by the cap curvature. This effective curvature is taken as the average of the two major radii of curvature which in turn are

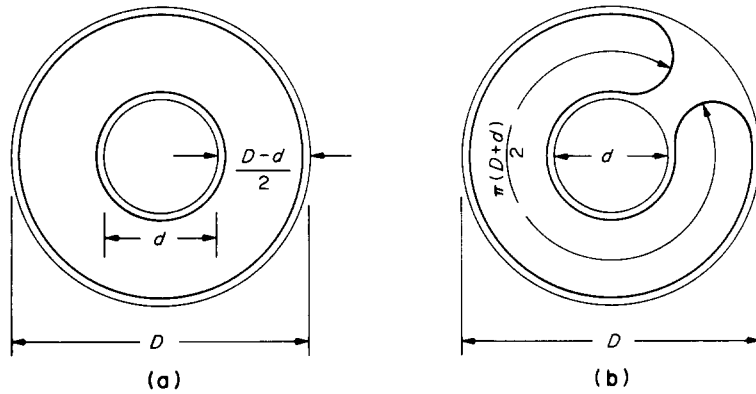


Figure 1. Bubble shape in an annulus.

estimated as the largest and shortest dimensions of the horizontal cross-section of the bubble. For rectangular ducts with sides a and b the effective diameter in [1] is thus $D_e = (a + b)/2$.

Consider now a large gas bubble rising in an annulus with an outer diameter D and inner diameter d . It was observed that the rise velocity in this case exceeds that of bubbles rising in a circular pipe with the same outer diameter. This result seems to be unexpected and puzzling. Furthermore, if a symmetrical ring bubble is assumed to exist in an annulus (figure 1a), the curvature diameter at the cap is of the order of $(D - d)/2$. Using this dimension as the characteristic diameter in [1] results in a bubble rise velocity which is considerably lower than that observed experimentally. Close observations show that, due to instability, the bubbles in annuli are not symmetrical (Griffith 1964), but rather have the form of an open annular ring, as shown in figure 1b. This open annular ring can be regarded as a wrapped rectangular bubble with two main characteristic curvature diameters: the long one is of the order of $\pi(D + d)/2$, while the short one is of the order of $(D - d)/2$. The resulting average diameter of curvature is given by

$$D_e = \frac{1}{2} \left[\frac{\pi(D + d)}{2} + \frac{(D - d)}{2} \right] = \frac{D}{4} \left[(\pi + 1) + \frac{d}{D} (\pi - 1) \right]. \quad [2]$$

One can expect thus that for annuli with a narrow gap ($d \approx D$), where the effective diameter according to [2] is $D_e = \pi/2 D > D$; the rising velocity will be determined by the outer diameter and will be higher than in the circular pipe of the same diameter.

In more complex geometries (e.g. rod bundles) explicit information regarding the resulting bubble shape seems to be needed in order to predict the rising velocity.

Using the above-suggested characteristic dimension, the validity of [1] was checked for available data for the rise velocity of single bubbles in rectangular channels and annuli.

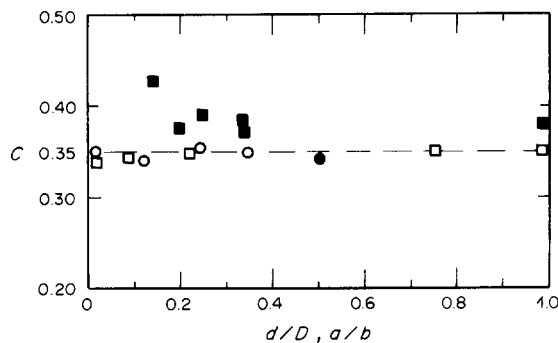


Figure 2. Experimental results for C using the proposed effective diameter. Dimension ratio: d/D for annulus and a/b for rectangular. Closed symbols, Sadatomi *et al.* (1982); open symbols, Griffith (1964); \square , \blacksquare , rectangular channel; \circ , \bullet , annulus.

The resulting constant C is shown in figure 2. Griffith's (1964) data correlates with C in a very narrow range of 0.34–0.36. The data of Sadatomi *et al.* (1982) for annuli correlates very well with the value of $C = 0.34$. Their data for rectangular channels result in somewhat higher but still reasonable values of C .

It should be emphasized that the main purpose of the present note is not only to come up with a better suggestion for the characteristic dimension but rather to give some physical justification for the choice of the proper equivalent diameter, and especially to explain the surprising phenomena that bubbles in annuli rise faster than those in a tube with the same shroud diameter. The present approach provides a reasonable method to estimate the bubble rise velocity, provided experimental information on the actual bubble shape is available. This shape is determined by the duct geometry as well as by prevailing flow conditions.

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